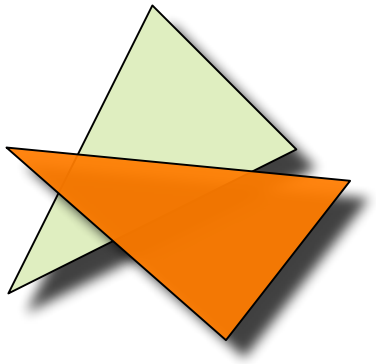
Lesson Plan Title: Congruent Triangles—Is This Enough Information?

Local School District: Western Local Schools

Grade Level: 7th Grade

Teacher Name: A. Dotson



Estimated Total Time: 172 minutes
Daily Time Allocation: 43 minutes
Duration: 4 sessions

Goal: Students will learn about some of the congruence postulates and apply the knowledge to triangles in the world around them.

Students will be able to do the following:

- Determine whether two triangles are congruent and provide the correct reason as to why the triangles are congruent.
- Name congruent triangles and provide the correct reasoning for the triangles being congruent.
- Use the fact that all of the corresponding parts are congruent to solve new problems.
- State the information that is required in order for the triangles to be congruent.

Outcomes: Students will use the Geometer's sketchpad to apply the concepts of congruency measure angles. Students collect evidence of congruent angles in the world.

National Standards:

National Council for Teachers of Mathematics (NCTM) **(http://standards.nctm.org/document/chapter5/index.htm)**
In grades 6–8 all students should:

- Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.
- Create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.
- Use geometric models to represent and explain numerical and algebraic relationships.
- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

International Society for Teacher Educators (ISTE) Performance Indicators For Technology—Literate Students (NETS) (http://cnets.iste.org/students/s_profile-68.html

Use content-specific tools, software, and simulations (e.g., environmental probes, graphing calculators, exploratory environments, Web tools) to support learning and research.

Select and use appropriate tools and technology resources to accomplish a variety of tasks and solve problems.

Ohio Standards Connections:

- **Key Standard(s):**
  Geometry and Spatial Sense Grades
  5 –7, F. Describe and use the concepts of congruence, similarity, and symmetry to solve problems.
- 5 –7, J. Apply properties of equality and proportionality to solve problems involving congruent and similar figures (e.g., create a scale drawing).

Grade Level Indicator(s):
Geometry and Spatial Sense:
7-4. Determine necessary conditions for congruence of triangles.
7-5. Apply properties of congruent and similar figure to solve problems involving missing lengths and angle measures.

Mathematical/Scientific Processes:
7-E. Use deductive reasoning to construct informal arguments to support reasoning and to justify solutions to problems.
7-K. Recognize and use mathematical language and symbols when reading, writing, and conversing with others.

- **Other Related Benchmarks:**

Measurement
7-C. Identify appropriate tools and apply appropriate techniques for measuring angles, perimeter or circumference, and area of triangles, quadrilaterals, circles, and composite shapes, and surface area, and volume of prisms and cylinders.
7-D. Select a tool and measure accurately to a specified level of precision.

Preassessment:

Concepts students should know before this lesson is taught:

- Symbol for congruence (≅)
- Congruence of Triangles (Corresponding Parts Congruent)
- If Δ ABC ≅Δ DEF, list the corresponding parts
- [∠A ≅∠D,∠B ≅∠E,∠C ≅∠F,$AB \cong DE$,$BC \cong EF$,and $AC \cong DF$]
- Measuring angles
- Drawing angles of specified measure
- Sum of the angles of a triangle is 180 °
- Terms students will need to be familiar with:
- Vertex (vertices)
- Midpoint
- Vertical Angles
- Right Triangle
- Hypotenuse

Strategy: Class discussion about the terms, quick quiz to determine mastery level of terms.

Postassessment:

Multiple-Choice Quiz

Scoring Criteria:

4-point rubric

1 point: drawing Δ IJK and Δ QRS and marking the given information
1 point: Δ JKI ≅Δ RSQ
1 point: Triangles are congruent because of ASA.
1 point: $IK \cong QS$ because of CPCTC.

Material and Resources Needed:

☐ Geometer's Sketchpad
☐ Overhead projector
☐ Computer lab
☐ Rulers
☐ Protractors
☐ Brand new pencils
☐ Board protractor
☐ SSS, SAS, ASA Triangle Congruence Worksheet (problems taken from *Informal Geometry* Lesson 5-5)
☐ AAS and HL Triangle Congruence Worksheet (problems taken from *Informal Geometry* Lesson 5-8)
☐ Applying Triangle Congruence Worksheet (idea from *CORD Geometry**)
☐ Showing Triangles Congruent Worksheet (problems taken from *Informal Geometry**Lesson 5-6)
☐ Corresponding Parts Congruent Worksheet (problems taken from *Informal Geometry**Lesson 5-7)

Sources:

Cord. (1999). *Cord geometry. Upper* Saddle River, NJ: Globe Fearon.

Smith, S. A., Nelson, R. W., Koss, R. K., Keedy, M. L., Bittinger, M. L., & Smith, R. T. (1992). *Informal geometry.* Boston: Addison-Wesley.

Procedures:

Day 1:

1. Pose the problem situation:
   Prior to the start of a yacht race, you (the judging official) must certify that all of the sails are the same size. Without unrigging the triangular sails from their masts, how can the official (you) determine if the sails on each of the boats are the same size?
2. Allow students to answer in small groups discussing how they came up with the solution.
3. A spokesperson for each group will present the team's solution.
4. Explain that over the next couple of days we (the class) will be learning some geometry tricks (concepts) involving triangles that will help us answer the above question.
5. Give each student a ruler, a protractor, and a brand new pencil.
6. Using these three objects, each student will create a triangle (the three sides being the ruler, the unsharpened pencil, and the straightedge of the protractor).
7. Once the triangles are created, have the students compare triangles with their teams.
8. Ask if everyone created exactly the same triangle? (Yes.)
9. Choose two students' triangles to share with the class and compare.
10. Lay one triangle on top of the other, showing that all of the parts are exactly the same measure.
11. Discuss with the class that the triangles are congruent: all of the parts are the same or congruent.
12. Note to the class that we didn't even pay any attention to the angles and they "took care of themselves."

    This is called SSS (Side-Side-Side) Congruence —all one has to have is the sides congruent on two triangles and the triangles are congruent because only one triangle can be formed from the given information.

13. Each student can create another triangle using the three objects, but this time only using 8 or 9 inches of the ruler for one of the sides.
14. Discuss the findings, and then relate back to the initial yacht problem: How could the official make sure that the sails are the exactly the same?
    The official could use SSS Congruence —make sure all of the sides are congruent.

Day 2:

1. In groups of three or four students, the students can complete the following activity.
2. Teacher models the steps on the board using the teacher board protractor.

    a. Draw a 6-inch segment.
    b. Label it *GH*.
    c. Using your protractor, make ∠G =60.
    d. From vertex G, draw *GI* measuring 7 inches long.
    e. Label the end point I.
    f. Ask the class from the given information, how many different triangles can be formed?
    (One)
    g. Form Δ GHI.
    h. Discuss the information used to create this triangle.
    (SAS Side –Angle –Side).
    i. Complete similar steps to show ASA (Angle –Side –Angle) Congruence.
3. Once the class demonstrates competency working with triangles using SAS and ASA, give the students a worksheet reviewing SSS, SAS, and ASA Congruence of Triangles.
4. The students will need to determine why the triangles are congruent.
5. Complete several of the problems on the worksheet together as a class
6. Review the answers.
7. Assign the rest for homework.
8. Refer back to the "Yacht Problem." How could the official make sure that the sails are the exactly the same?
    a. SSS Congruence —make sure all of the sides are congruent.
9. How else could the official check the sails?
    a. SAS and ASA Congruence
10. In teams, have the students discuss how the official would use SAS and ASA Congruence. Which method is the most logical (easiest/quickest) for the official?
11. The team member can report the team's suggestions.

Day 3:

1. Review with the class the SSS, AS, SA Congruence homework.
2. Ask for questions.
3. Collect paper for review and assessment.
4. Introduce two "tricks " for reasoning triangles congruent:
    a. AAS (Angle –Angle –Side) Congruence
    b. HL (Hypotenuse –Leg) Congruence.
5. Explain AAS.
6. Explain HL to the class.
7. Demonstrate examples on the board.
8. Distribute the AAS and HL worksheet.
9. Provide in class time for students complete the worksheet working in pairs.

10. Students will need to be able to explain in full sentences to each other why the given triangles are congruent.
11. Spot check for mastery.

Day 4:

1. Using the computer lab or a classroom for pairs of students to work on the computer, allow students to launch Geometer's Sketchpad on the computer.
2. The teacher should demonstrate on the overhead how to draw an object on the screen and make a copy that is rotated.
3. Elicit from the students the characteristics of the two objects attempting to draw out that the angles and length of sides are the same regardless of rotation.
4. Draw two more objects and have the students recognize congruent angles or sides and support their decisions.
5. Ask students to review isosceles triangles and create one on their screen.
6. Have students create a polygon and flip it or rotate it by either marking a center or by using the mirror option.
7. With an object rotated or flipped, ask students to identify congruent sides or angles on their polygon and its image.
   Congruence is often used in architecture and in art.
8. Distribute the Applying Triangle Congruence Worksheet.
9. Allow students to work with a different partner from the previous lesson to complete the worksheet.
10. Review the results as a whole class.
11. Distribute the Showing Triangles Congruent Worksheet.
12. Complete the first problem as a whole group.
13. Students can work in pairs to complete the remaining problems.
14. Review/discuss it together in small groups.
15. Review with congruent triangles.
16. Ask students to come to the board to illustrate two or more examples.
    Students need to mark necessary information so that the triangles will be congruent.
17. Explain to the students that these are examples of the concept Corresponding Parts of Congruent Triangles are Congruent, which we will call CPCTC.
18. Distribute the students the Corresponding Parts Congruent Worksheet.
19. Discuss/review the first couple problems, and then have the students complete the rest in pairs.
20. Review and discuss the worksheet.
21. Collect the worksheet for further assessment.

Differentiated Instruction Strategies: (http://www.teach-nology.com/tutorials/teaching/differentiate/bottom_line/)

Enrichment: Some students may be interested in why AAS and HL Congruence works.

For AAS: Given: Δ VWU and Δ DEC where ∠W ≅∠C, ∠U ≅∠D, and VU ≅DE

1. Draw these two triangles on the board.
2. Students draw the triangles and mark the given information of the triangles at their seats.
   Since the triangles have two angles congruent, the third angle would have to be congruent.
3. Have *m* ∠W =60 ° and *m* ∠U =40 °.
   The measure of ∠V would have to be 80 °, because there are 180 ° in a triangle.
   m ∠C would have to be 60 ° because ∠W ≅∠C and m ∠D would have to be 40 ° because ∠U ≅∠D
   m ∠E =80 °
   Since ∠V ≅∠E, Δ WUV ≅Δ CDE because of ASA
   ****AAS is just a shortcut for ASA.*
4. Students can do other examples to show that AAS is always true.
5. Use these same examples and have students suggest the measures of the two given angles.
   The measures of the angles can be anything as long as the triangle has 180 °.
6. To conduct further work on HL, students will need the ruler, protractor, and compass.
7. Students can draw a horizontal segment *ST* at least 3 inches long.
   **Model this for the students on the board.
8. Work through this example together.
9. Draw a 2-inch segment *SR* at a 45 ° angle from S.
10. Open the compass so that the compass point is at R and the pencil point just across *ST*.
11. Draw an arc that intersects *ST* in as many points as possible.
12. How many points (intersections) did the arc made with *ST*.
13. Label the intersection points U and V.
14. Form Δ SRU and Δ SRV.
15. Discuss with the class if Δ SRU ≅Δ SRV.
    They are not.
16. Is SSA (the information we used to create these triangles) sufficient to show triangles congruent?
    No.
17. Repeat the above steps, but this time, use a 90 ° angle.
18. Discuss with the class that SSA only works for right triangles thus the name Hypotenuse-Leg.

Extension:

1. Show students an Escher tessellation drawing and elicit the congruence that exists.
2. Ask to point out congruent angles that exist in the room. Students should recognize that there are many examples of congruent angles and sides. They can use the digital camera to record images of congruent triangles. The pictures can be collected in a PowerPoint® presentation.

Using Geometer's Sketchpad:

3. Create two or more figures using segments, arcs, circles, or whatever. The figures can be letters, geometric figures, or even caricaturized faces. Now, using Movement buttons (or, for more control and sophistication, a slider controlling a dilation from one state to the other), set things up so that one figure "morphs" into the other.
4. Construct a circle and an inscribed angle. Measure the angle and the arc it intercepts. Animate the vertex of the inscribed angle to demonstrate that every inscribed angle that intercepts this arc has the same measure.

Homework Options and Home Connections:

1. Worksheets completion:
   - SSS, SAS, ASA Triangle Congruence Worksheet
   - AAS and HL Triangle Congruence Worksheet
   - Applying Triangle Congruence Worksheet
   - Showing Triangles Congruent Worksheet
   - Corresponding Parts Congruent Worksheet
2. Take digital pictures of triangles in the house. Identify how each of these congruencies could help a builder to construct furniture.

Key Vocabulary:

- SSS (Side –Side –Side) Congruence
- SAS (Side –Angle –Side) Congruence
- ASA (Angle –Side –Angle) Congruence
- AAS (Angle –Angle –Side) Congruence
- HL (Hypotenuse –Leg) Congruence
- CPCTC (Corresponding Parts of Congruent Triangles are Congruent)

Technology Tips:

Geometer's Sketchpad should be installed on each of the computers prior to the lessons.

> *Geometer's sketchpad is Geometry Software for Exploring Mathematics. Not only is it a mathematics visualization environment, but its construction and exploration tools enable students to explore and understand mathematics in ways that are simply not possible with traditional tools. Using Sketchpad will add a powerful dimension to this lesson.*

References:

Albrecht, Masha. (1996.) Integrating Algebra and Geometry with the Geometer's Sketchpad. Key Curriculum Press, 1996.

Principles and Standards for School Mathematics. (2000.) National Council of Teachers of Mathematics, Inc.

King, J., & Schattschneider. (1997.) Geometry Turned On: Dynamic Software in Learning, Teaching, and Research. (http://mathforum.org/dynamic/geometry_turned_on/) *The Mathematical Association of America* (http://www.maa.org/) 41, (206) p. 1997.

NAME: Preassessment

DATE: **Mathematic Terms**

What is the symbol for congruence?

Congruence of Triangles (Corresponding Parts Congruent):
If Δ ABC ≅Δ DEF, list the corresponding parts:
∠A ≅
∠B ≅
∠C ≅
$\overline{AB}$ ≅
$\overline{BC}$ ≅
$\overline{AC}$ ≅

What is one technique for measuring angles?

How can you draw angles of specified measure?

What is the sum of the angles of a triangle?

Terms students will need to be familiar with:

Using complete sentences, provide a thorough definition for each term or symbol.

Vertex (vertices):

Midpoint:

Vertical Angles:

Right Triangle:

Hypotenuse:

NAME: **Post Quiz**

DATE: **Congruency**

For Δ *ABC* ≅Δ *XYZ* and knowing that *AB* ≅*XY*, what else must we know in order to use SAS Congruence?

A .*BA* ≅*YX* ∠*B* ≅∠*Y*
B. *AC* ≅*XZ* ∠*C* ≅∠*Y*
C. *BC* ≅*XZ* ∠*C* ≅∠*Z*
D. *AC* ≅*XZ* ∠*A* ≅∠*X*

Short Answer:
Given:
∠C ≅∠N, ∠P ≅∠E, *DE* ≅*OP* (and accompanying diagram Δ CED and Δ NPO)
Then
Δ _≅Δ PNO by

Extended Response:
Given:Δ IJK and Δ SRQ where ∠J ≅∠R, ∠K ≅∠S, *RS* ≅*KJ*

Draw and mark the given information on the triangles and explain why *IK* ≅*QS* .